



**THIRD SPACE  
LEARNING**

# Math Intervention Pack

Understanding multiplication  
of a whole number by a non-  
unit fraction

**Grade 4**

## How To Use This Resource

### 1. Title Slide

Use this slide to activate prior knowledge needed for lesson. Students should be encouraged to initially attempt the question presented independently.

### 2. Let's Learn

Use this slide to introduce the concept. Tutors should work with the student to explore the concept together, usually using diagrams to support understanding.

### 3. Follow Me + Your Turn

The tutor should work through the follow me slide, modeling the process and explaining their thinking out loud.

Students should use the your turn slide as an opportunity to work through a question similar to the follow me questions. They should apply the method modeled by the tutor in the follow me slide. Students should be encouraged to explain their thinking out loud.

### 4. You Do

Students should work through a range of questions that build in complexity.

Tutors can offer support but students should initially be encouraged to attempt these questions independently.

### 5. Go Further

Use this slide to allow students to apply their understanding to a more challenging question in an unfamiliar context.

### 6. Support for Slides

The support slide is used to support students during the lesson. In the tutor notes, there will be guidance as to when to use the support slide.

### 7. Check Your Understanding

Tutors should use this slide to assess the student's knowledge and whether or not they have mastered the concept within the lesson.

## Standard

**4.NF.4b:** Understand a multiple of  $\frac{a}{b}$  as a multiple of  $\frac{1}{b}$ , and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express  $3 \times (\frac{2}{5})$  as  $6 \times (\frac{1}{5})$ , recognizing this product as  $\frac{6}{5}$ . (In general,  $n \times (\frac{a}{b}) = \frac{(n \times a)}{b}$ )

## Key Mathematical Ideas

1. Use a visual fraction model to represent multiplication of a whole number by a non-unit fraction
2. Use repeated addition to multiply a whole number by a non-unit fraction
3. Understand multiplication of a whole number by a non-unit fraction as multiples of the unit fraction

## Overview

### Terminology

- **Multiplication:** The repeated addition of equal groups
- **Fraction:** A part of a whole number
- **Non-unit fraction:** A fraction with a numerator greater than 1
- **Numerator:** The top number of a fraction which tells the number of equal-sized parts of the whole
- **Denominator:** The bottom number of a fraction which tells the number of equal-sized pieces in the whole
- **Common denominator/Like denominator:** Having the same denominator

### Sentence Stems

- ..... groups of .....
- $a \times \frac{b}{c} = \frac{a \times b}{c}$

## Overview

### Common Misconceptions

Common Misconceptions	Tutoring Strategies	Checks for Understanding
As students are using repeated addition as a strategy during this lesson, they may add the numerators and the denominators instead of keeping the denominators the same.	<p>Ask/remind students of the rules when we add fractions with the same denominator.</p> <p>Read the equation out loud for students. For example, “two-eighths plus two-eighths is how many eighths?” (four-eighths)</p>	How do we add fractions with the same denominator?

## Title Slide

### If stuck

- Have students count how many eighths are shown (9)
- Ask them - what is  $\frac{3}{8} + \frac{3}{8} + \frac{3}{8}$  ? ( $\frac{9}{8}$ )

Note that students can also write  $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{9}{8}$

- This strategy can be used throughout the remainder of the lesson, as well.

### Answers

- $\frac{9}{8}$



## Let's Learn

### If stuck

- Remind students that when we add fractions with the same denominators, we add the numerators and keep the denominators the same.

### Questions

- How are multiplying whole numbers and multiplying a whole number by a fraction similar?
- How is multiplying a whole number by a fraction similar to adding fractions?
- How would we write  $\frac{12}{3}$  as a mixed number? ( $\frac{12}{3}$  is equal to the whole number 4)

### Watch out for

- Students adding the numerators and the denominators - denominator should stay the same

### Answers

a.  $3 + 3 + 3 + 3 + 3 + 3 = 18$

b. 18

## Follow Me

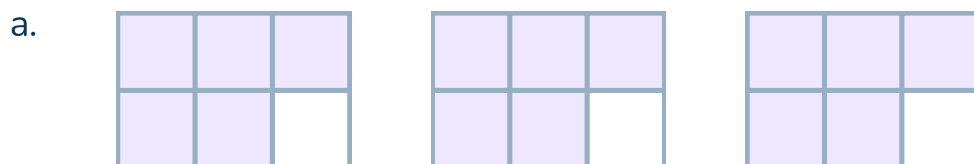
### Modeling prompts

- To partition these rectangles, I will first cut it into thirds by drawing two vertical lines so that the rectangle has 3 equal-sized sections. Then, I will cut the rectangle in half horizontally. This will give me 6 equal parts.
- Once I do this to all 3 rectangles, I will shade on out of every 6 parts on each rectangle.
- I know that each of the shaded parts equals  $\frac{1}{6}$  and each rectangle has  $\frac{5}{6}$  shaded.

I also know that this model shows 3 groups of  $\frac{5}{6}$ .

- To write this as a repeated addition equation, I will write  $\frac{5}{6} + \frac{5}{6} + \frac{5}{6}$ .  
To add these fractions together, we add only the numerators.  $5 + 5 + 5 = 15$ . The denominators stay the same - so the denominator is 6.
- Therefore,  $3 \times \frac{5}{6} = \frac{15}{6}$
- How would we write this answer as a mixed number? ( $\frac{15}{6}$  as a mixed number is  $2\frac{3}{6}$  or  $2\frac{1}{2}$ ).

### Answers



b.  $\frac{5}{6} + \frac{5}{6} + \frac{5}{6} = \frac{15}{6}$

c.  $\frac{15}{6}$

## Your Turn

### If stuck

- Help students create their visual model and write the addition equation to represent it as needed.

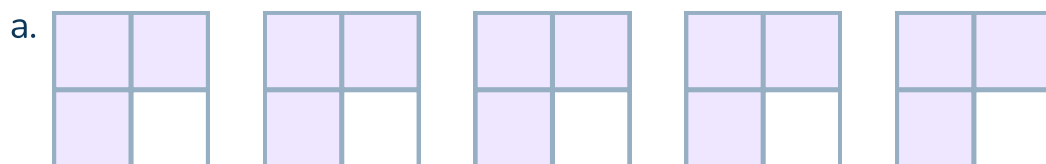
### Questions

- How does your visual model represent the multiplication equation?
- How does your repeated addition equation represent the model? How does it represent the multiplication equation?
- Why can we read the equation  $5 \times \frac{3}{4}$  as 5 groups of  $\frac{3}{4}$ ?
- How would we write this answer as a mixed number? ( $\frac{15}{6}$  as a mixed number is  $3 \frac{3}{4}$ )

### Watch out for

- Students adding the numerators and the denominators - denominator should stay the same

### Answers



b.  $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{15}{4}$

c.  $\frac{15}{4}$

## You Do

### If stuck

- Show students the Let's explore this more slide - this will show them another strategy to use (a number line) if they are struggling with this one.

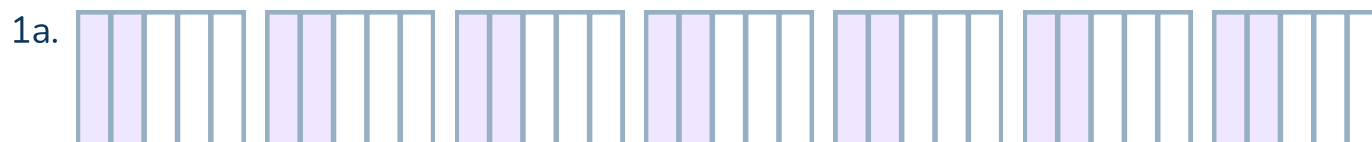
### Questions

- How did you know how many rectangles (use whatever shape they drew here) to draw?
- How did you know how many parts to partition each shape into?
- How does your model represent the equation?
- How does your addition equation compare to your multiplication equation?

### Watch out for

- Students drawing the wrong number of wholes or partitioning the wholes incorrectly.

### Answers



b.  $\frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{14}{5}$

c.  $7 \times \frac{2}{5} = \frac{14}{5} = 2 \frac{4}{5}$



b.  $\frac{3}{10} + \frac{3}{10} + \frac{3}{10} + \frac{3}{10} = \frac{12}{10}$

c.  $4 \times \frac{3}{10} = \frac{12}{10} = 2 \frac{2}{10}$

## Go Further

### If stuck

- Walk students through the word problem/situation and ask them to draw a picture of what it is describing - 8 pies with  $\frac{2}{3}$  of each pie eaten.

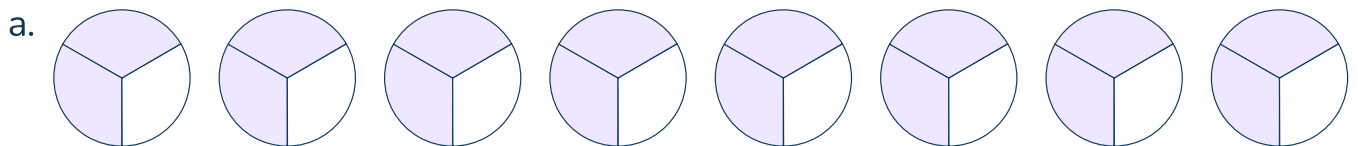
### Questions

- Why is it important that each pie is the same size? How does this affect our ability to accurately solve the problem? (In order to add or multiply fractions, they must be in reference to the same whole and the same size pieces.  $\frac{2}{3}$  of a very small pie and  $\frac{2}{3}$  of a very large pie are different sizes.)
- How can you write your answer as a mixed fraction? ( $5\frac{1}{3}$ )

### Watch out for

- Students drawing the wrong number of wholes or partitioning the wholes incorrectly.
- Students misinterpreting the word problem.

### Answers



b.  $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{16}{3}$

c.  $8 \times \frac{2}{3} = \frac{16}{3}$

As a mixed number:  $5\frac{1}{3}$

## Support for Slide(s)

This slide supports the Yo Do slide.

### If stuck

- Compare the number lines to the visual models
- Point out how each part of this strategy represents the equation

### Questions

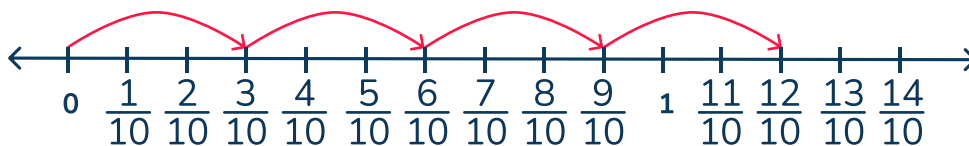
- What do the jumps represent in the equation? What does the number line represent in the equation?

### Answers

1b. 7 jumps

1c.  $\frac{14}{4}$  as a mixed number:  $2\frac{4}{10}$

2a. tenths



2b. 4

2c.  $\frac{12}{10}$  as a mixed number:  $1\frac{2}{10}$

## Check Your Understanding

### Correct answers

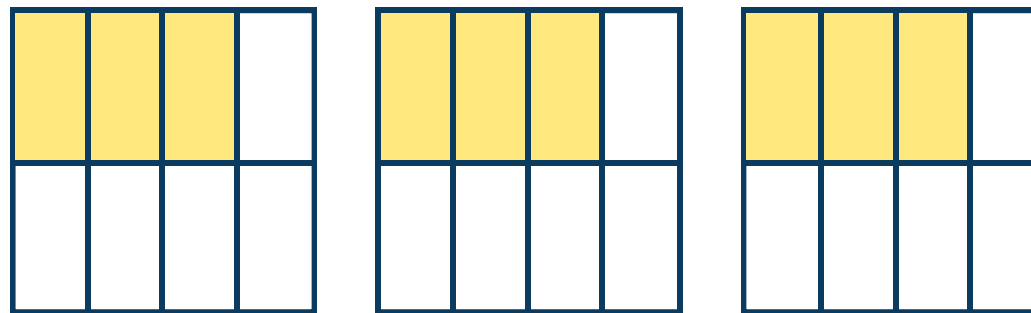
- d.  $\frac{12}{5}$
- a. Students may choose this answer if they switch the numerator and denominator.
- b. Students may choose this answer if they multiply 3 by both the numerator and denominator of the fraction OR if they write a repeated addition equation and add the numerators and the denominators instead of keeping the denominators the same.
- c. Students may choose this answer if they add the whole number and the numerator instead of multiplying.
- d. This is the correct answer. Students can add  $\frac{4}{5}$  together 3 times, which is  $\frac{12}{5}$ .

Today you will learn about

# Understanding multiplication of a whole number by a non-unit fraction

Warm-up question

How many eighths are shown?



Write your answer as a fraction.

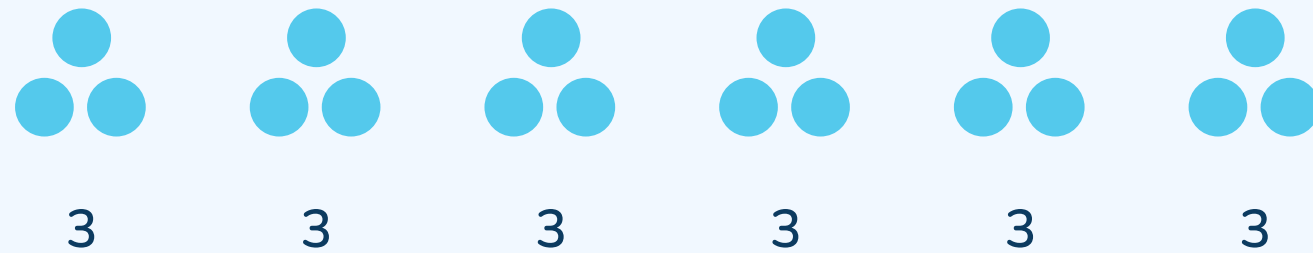


# Let's learn

We can use **visual models** and **repeated addition** to multiply a whole number by a non-unit fraction, similar to the way we learned to multiply whole numbers.

$$6 \times 3$$

We think of this as 6 groups of 3



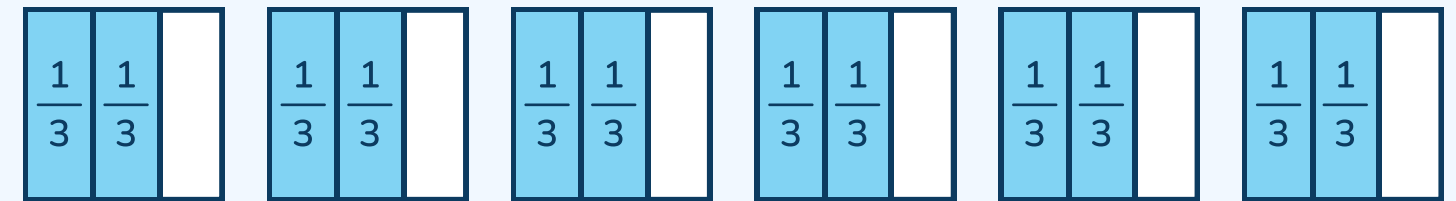
a Write a repeated addition equation to represent the model.

..... + ..... + ..... + ..... + ..... + ..... = .....

b So  $6 \times 3 =$  .....

$$6 \times \frac{2}{3}$$

We think of this as 6 groups of  $\frac{2}{3}$



a Write a repeated addition equation to represent the model.

$\frac{\boxed{\phantom{000}}}{\boxed{\phantom{000}}} + \frac{\boxed{\phantom{000}}}{\boxed{\phantom{000}}} + \frac{\boxed{\phantom{000}}}{\boxed{\phantom{000}}} + \frac{\boxed{\phantom{000}}}{\boxed{\phantom{000}}} + \frac{\boxed{\phantom{000}}}{\boxed{\phantom{000}}} + \frac{\boxed{\phantom{000}}}{\boxed{\phantom{000}}} = \frac{\boxed{\phantom{000}}}{\boxed{\phantom{000}}}$

b So  $6 \times \frac{2}{3} = \frac{\boxed{\phantom{000}}}{\boxed{\phantom{000}}}$

How would you write the answer as a **mixed number**?

## Follow me

Solve  $3 \times \frac{5}{6}$

- a Partition each rectangle into sixths Shade in  $\frac{5}{6}$  of each rectangle.



- b Write a repeated addition equation to represent the model

$$\frac{\boxed{\phantom{000}}}{\boxed{\phantom{000}}} + \frac{\boxed{\phantom{000}}}{\boxed{\phantom{000}}} + \frac{\boxed{\phantom{000}}}{\boxed{\phantom{000}}} = \frac{\boxed{\phantom{000}}}{\boxed{\phantom{000}}}$$

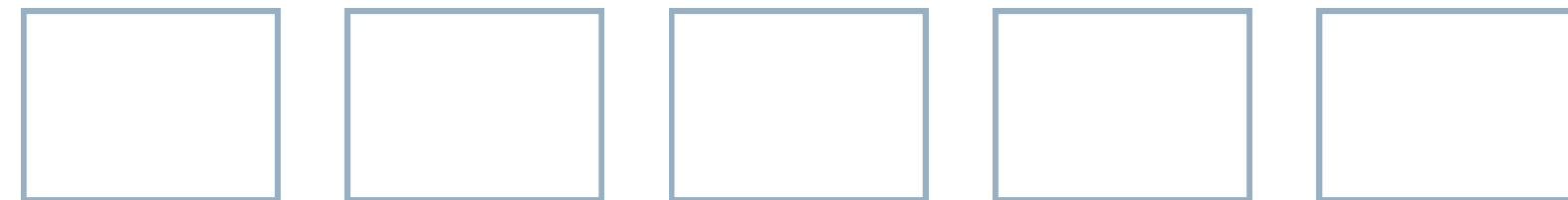
c  $3 \times \frac{5}{6} = \frac{\boxed{\phantom{000}}}{\boxed{\phantom{000}}}$

How would you write the answer as a **mixed number**?

## Your turn

Solve  $5 \times \frac{3}{4}$

- a Partition each rectangle into fourths. Shade in  $\frac{3}{4}$  of each rectangle.



- b Write a repeated addition equation to represent the model

$$\frac{\boxed{\phantom{000}}}{\boxed{\phantom{000}}} + \frac{\boxed{\phantom{000}}}{\boxed{\phantom{000}}} + \frac{\boxed{\phantom{000}}}{\boxed{\phantom{000}}} + \frac{\boxed{\phantom{000}}}{\boxed{\phantom{000}}} + \frac{\boxed{\phantom{000}}}{\boxed{\phantom{000}}} = \frac{\boxed{\phantom{000}}}{\boxed{\phantom{000}}}$$

c  $5 \times \frac{3}{4} = \frac{\boxed{\phantom{000}}}{\boxed{\phantom{000}}}$

How would you write the answer as a **mixed number**?

## You do

1  $7 \times \frac{2}{5}$

a Draw a model to represent the expression.

b Write a repeated addition equation.

c Write a multiplication equation.

How would you write the answer as a mixed number?

2  $4 \times \frac{3}{10}$

a Draw a model to represent the expression.

b Write a repeated addition equation.

c Write a multiplication equation.

How would you write the answer as a mixed number?

## Go further

Ramona's aunt served eight pies after a family dinner.

So far,  $\frac{2}{3}$  has been eaten from each pie .

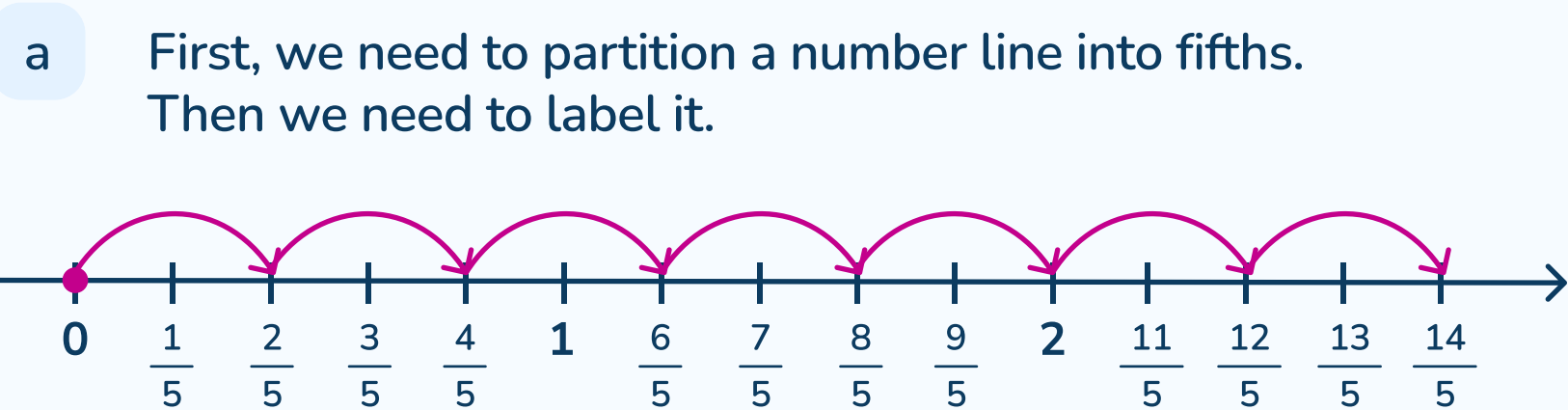
How much pie has been eaten altogether?

- a Draw a model of the pies. Shade in the portion that has been eaten from each one.
- b Write and solve a repeated addition equation to represent the model.
- c Write and solve a multiplication equation to represent the problem.

Support

We can also multiply a whole number by a fraction using a number line.

1      $7 \times \frac{2}{5}$

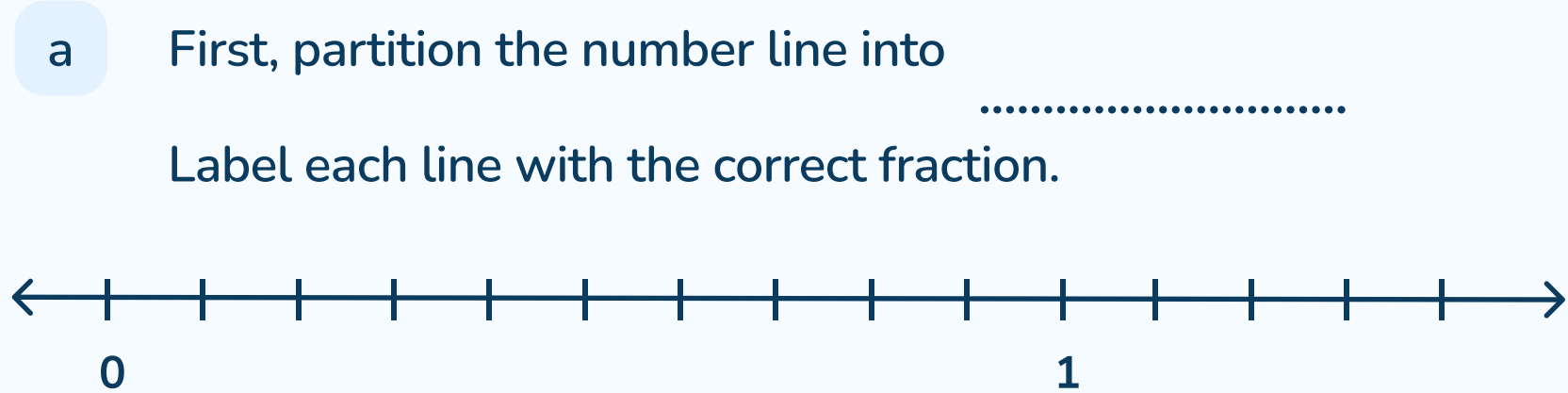


b     After you partition the number, start at zero and make **7 jumps**, since we need to show  $\frac{2}{5}$  **7 times**. Each jump should cover  $\frac{2}{5}$

b     So,  $7 \times \frac{2}{5} = \frac{\boxed{\dots\dots\dots}}{\boxed{\dots\dots\dots}}$

How would you write the answer as a **mixed number**?

2      $4 \times \frac{3}{10}$



b     Start at zero, then make ..... jumps.  
Draw the jumps on the number line above.

c     The fraction that you land on is the answer.

So,  $4 \times \frac{3}{10} = \frac{\boxed{\dots\dots\dots}}{\boxed{\dots\dots\dots}}$

How would you write the answer as a **mixed number**?

## Check your understanding

$$3 \times \frac{4}{5} =$$

a

$$\frac{5}{12}$$

b

$$\frac{12}{15}$$

c

$$\frac{7}{5}$$

d

$$\frac{12}{5}$$

Why do I need to try this question on my own first?

- To show your tutor what you understand
- To give you more practice
- To show your teacher how you are doing



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- ✓ Aligned to your state's standards
- ✓ Scaffolded learning to close gaps

“We just had our first session and it went great! The kids really liked it and felt like they were learning! One even said he finally felt like math was making sense.”



Michelle Craig, Instructional Coach,  
Sherwood Forest Elementary, Washington

## Speak to us

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